UNIVERSITY POLITEHNICA OF BUCHAREST

Center for Research and Training in Innovative Techniques of Applied Mathematics in Engineering *Traian Lalescu* (CiTi)

&

"1 DECEMBRIE 1918" UNIVERSITY OF ALBA IULIA

BOOK OF ABSTRACTS
ISIM & ISWIM

International Symposium & International Student Workshop on Interdisciplinary Mathematics in the CiTi areas

Volume I

ABOUT CiTi

Center for Research and Training in Innovative Techniques of Applied Mathematics in Engineering "*Traian Lalescu*" (CiTi), organized within the Faculty of Applied Sciences of the University POLITEHNICA of Bucharest, was established to develop and promote some innovative engineering mathematical techniques with a large application in different areas.

CiTi is open to students in the bachelor, master and doctoral programs from all types of faculties, with the aim of training and stimulating student research, as well as scientific collaboration between all relevant institutions both in the country and abroad, based on bilateral agreements in national, respectively in international programs/projects.

The main areas of research are:

- **Fractional calculation and applications**
- **Evolutionary algorithms and applications**
- **Fractional calculus, wavelet analysis and applications**
- **Generalized rough sets, hybridizations and new approximation spaces**
- **Innovative game theory techniques and applications**
- **Data analysis and information security in electronic format**
- **Monitoring the structural health of living organisms and environmental materials**
- **Smart Grids Vs. BlockChains**
- **New security technologies based on hardware devices and post-quantum cryptography**

Link access: https://citi.upb.ro/en/

ABOUT ISIM & ISWIM 2022 - 1st Edition

International Symposium & International Student Workshop on Interdisciplinary Mathematics in the CiTi areas ISIM & ISWIM 2022 – 1st Edition (www.isimconference.eu) was organized by University POLITEHNICA of Bucharest, Center for Research and Training in Innovative Techniques of Applied Mathematics in Engineering "*Traian Lalescu*" (CiTi) in collaboration with "1 Decembrie 1918" University of Alba Iulia, Romania.

The scope of the Symposium included but was not limited to original research works and ideas related to CiTi interdisciplinary areas and Applications: Fractional Calculus, Wavelet Analysis, Evolutionary Algorithms, Data Analysis and Information Security, Game Theory - including Quantum, Partial and Ordinary Differential Equations, Mathematical Statistics, Graph Theory.

The first edition was dedicated in memoriam of the 140 years from the birth of Traian Lalescu, The outstanding personality of the Romanian school of mathematics and engineering, founder of higher education institutions University Politehnica of Timisoara and Polytechnic Institute of Bucharest. The biography of Traian Lalescu is a dense biography focused in far too few years of life - only 47.

During his life Traian Lalescu published a whole series of articles and scientific books in various fields: functional equations, trigonometric series, mathematical physics, mechanical geometry, algebra or the history of mathematics.

ISIM SECTIONS

1. Fractional calculus, wavelet analysis and applications, evolutionary algorithms and applications.

2. Data analysis and information security, game theory - including quantum and applications.

3. CiTi related interdisciplinary areas and applications: Partial and Ordinary Differential Equations, Mathematical Statistics, Graph Theory.

ISWIM SECTIONS

1. Applications of fractional calculus.

2. Applications relative to data analysis and security.

3. Applications related to CiTi interdisciplinary areas: Partial and Ordinary Differential Equations, Mathematical Statistics, Graph Theory.

With support from the organizing institutions, participation to the First Edition of the ISIM & ISWIM was FREE OF CHARGE.

PAPER PUBLICATION

The most scientifically valuable works have been invited for publication in one of the following WOS indexed international journals:

UPB Scientific Bulletin, Series A: Applied Mathematics and Physics

(JIF 2020 : 0.903, JCR Categories: Applied Mathematics; Multidisciplinary Physics, WoS, Q4)
https://www.scientificbulletin.upb.ro

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*Selected papers from the conference can enjoy a 20% discount for the full paper publishing fee in the two MDPI journals - Special Issues.

We had invited prestigious **Keynote Speakers**:

Ravi P. AGARWAL, Texas A&M University, Kingsville, USA

Dumitru BĂLEANU, Cankaya University, Ankara, Turkey

Maslina DARUS, Universiti Kebangsaan Malaysia, Selangor, Malaysia

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Carlo CATTANI, University of Tuscia, Viterbo, Italy

Anjan BISWAS, Alabama A&M University, USA

Prof. Dr. Dorota MOZYRSKA - Bialystok University of Technology, Poland

Dorota MOZYRSKA is an employee of the Department of Mathematics at the Faculty of Computer Science of the Bialystok University of Technology. She graduated in 1992 at the Branch of the University of Warsaw in Białystok, majoring in mathematics. Doctorate in mathematical sciences awarded in 2002 by the Council of the Faculty of Mathematics and Information Sciences of the Warsaw University of Technology. The habilitation degree in the field of automatics and robotics was awarded by the resolution of the Council of the Faculty of Electronics and Information Technology of the Warsaw University of Technology on September 23, 2014. Research interests: mathematical control theory, difference and differential calculus of fractional order, population dynamics. She is a co-author of about 107 publications. Dorota Mozyrska conducted or conducts classes in the following subjects: mathematics, basics of mathematics, applications of mathematics in environmental engineering, linear algebra, differential equations, mathematical analysis, mathematical statistics, probability calculus, descriptive statistics, econometrics, operations research, English-language courses under the Erasmus program and as visiting professor at African University of Science and Technology: mathematical statistics, advanced linear algebra, differential equations with Matlab, fractional calculus. He belongs to the associations: Polish Mathematical Society and the International Society of Difference Equations. Private interests (without priority order): bicycle, mountains, travels, movies, yoga, logical and board games, cats, poetry, music, social media, minerals, Biebrza and Bialowieza Forest, popularization of exact sciences, especially mathematics.

Title: *Fractional-variable-order digital controller design tuned with the chaotic yellow saddle goatfish algorithm for the AVR system*

Prof. Dr. Antonio LIOTTA - Free University of Bozen-Bolzano, Italy

Antonio LIOTTA (https://bit.ly/UniBzLiotta) is Full Professor at the Faculty of Computer Science, Free University of Bolzano (Italy), where he teaches Data Science and Machine Learning. Antonio's passion for artificial intelligence, has driven his academic career through the meanders of artificial vision, e-health, intelligent networks and intelligent systems. Antonio's team is renowned for his contributions to micro-edge intelligence and miniaturized machine learning, which have significant potential in harnessing data-intensive systems, for instance in the context of smart cities, cyber-physical systems, Internet of Things, smart energy, and machine learning with humans in the loop. He has led the international team that has recently made a breakthrough in artificial neural networks, initiating a new research strand on "sparse neural networks for embedded learning" (doi.org/10.1038/s41467-018-04316-3). Antonio was the founding director of the Data Science Research Centre at the University of Derby. He has set up several cross-border virtual teams, and has been credited with over 350 publications involving, overall, more than 150 co-authors. Antonio is Editor-in-Chief of the Springer Internet of Things book series (springer.com/series/11636), and associate editor of several prestigious journals. He is co-author of the books "Networks for Pervasive Services: six ways to upgrade the Internet" (springer.com/gp/book/9789400714724) and "Data Science and Internet of Things" (springer.com/gp/book/9783030671969).

Title: *Data Analysis for the Intelligent Internet of Things*

Prof. Dr. Maslina DARUS - Universiti Kebangsaan Malaysia, Selangor, Malaysia

Prof. Dr. Maslina DARUS (Universiti Kebangsaan Malaysia, Selangor, Malaysia) - Maslina Darus obtained her first degree in Mathematics from Acadia University, Nova Scotia in 1992. She obtained her PhD from University of Wales, Swansea, U. K. in 1996. She became a lecturer at Universiti Kebangsaan Malaysia and was promoted as Associate Professor in 2001. She was promoted to Full Professor in September 2006. She was appointed as Head of Mathematics programme from 2004-2009. Then from 2010 until 2013, she was the Chair of the School of Mathematical Sciences and later as Deputy Dean, Undergraduate & Alumni of Faculty of Science and Technology (2013-2018). She is very active in the Malaysian Mathematical Sciences Society (PERSAMA) and served as the Honorary Secretary for about 17 years (1997-2013). Currently, she is the President of the Malaysian Mathematical Sciences Society (PERSAMA), 2013-2023. Her interests in geometric function theory mainly focus on the properties of certain subclasses of univalent functions. She has graduated more than 30 PhD students. The results from her research have indeed been published in more than 700 articles in various refereed/indexed journals. She has collaborated with many mathematicians throughout the globe.

Title: *Generalized Differential Operators: Is constructing new operators worth it?*

Prof. Dr. Dumitru BĂLEANU - Cankaya University, Ankara, Turkey

Dr. Dumitru BĂLEANU was born December 4, 1964 in Romania. He completed his undergraduate at Craiova University, Faculty of Science, Physics Department in 1988. Dr. Dumitru Baleanu got the PhD in Physics in 1996 from the Institute of Atomic Physics, Romania. Since 2000, he has been a visiting staff member at the Department of Mathematics, Faculty of Arts and Sciences, Çankaya University, Ankara, Turkey since 2000. He is a professor at the Institute of Space Sciences from Magurele-Bucharest, Romania.His fields of interest include the fractional dynamics and its applications in science and engineering, fractional differential equations and their applications, discrete mathematics, control theory, mathematical physics, solitons theory, Lie symmetry, dynamic systems on time scales, the wavelet method and its applications, quantization of systems with constraints, the Hamilton-Jacobi formalism, geometries admitting generic and non-generic symmetries.

He is an editorial board member of several prestigious international journals as Applied Numerical Mathematics, Mathematics, Symmetry, Mathematical Methods in Applied Sciences, Advances in Difference Equations, Advances in Mechanical Engineering, Open Physics, Journal of Computational and Nonlinear Dynamics and Fractional Calculus and Applied Analysis. He is the chief editor of Frontiers in Physics-section Mathematical Physics and Progress in Fractional Differentiation and Applications. His h-index is 56 and he has been a highly cited researcher in Mathematics since 2015. He co-authored several published books in World Scientific Publishing, Academic Press, Springer and De Gruyter. He got several international prizes as Obada Prize (2019), ICFDA2018 Award: Innovation in Fractional Calculus and ICFDA 2016 Award: Dissemination Award.

Title: *Fractional Generalised Operators:Theory and Applications*

Prof. Dr. Carlo CATTANI - University of Tuscia, Viterbo, Italy

Carlo CATTANI is Professor (Habil. Full Professor, since 2017) of Mathematical Physics and Applied Mathematics at the Department of Economics, Engineering, Society and Enterprise (DEIM) of Tuscia University (VT)-Italy, since 2015. He has been previously appointed as professor/research fellow at the Dept. of Mathematics University of Rome "La Sapienza" (1980-2004) and Dept. of Mathematics, University of Salerno (2004-2015). Research fellow at the Italian Council of Research, CNR in 1978-1980 and Visiting Research fellow at the Physics Institute of the Stockholm University (1987-1988). His main scientific research interests is focussing on numerical and computational methods, mathematical models and methods, time series and data analysis, computer methods and simulations. Author of more than 300 scientific papers on international journals and co-author of several books he has given significant contributions to fundamental topics such as numerical methods, dynamical systems, fractional calculus, fractals, wavelets, nonlinear waves, data analysis, neural networks and artificial intelligence. Editor in Chief of the journals "Fractal and Fractional" and "Information Sciences Letters", he is serving also as Editor in Several International Scientific Journals. He has been awarded Honorary professor at the Azerbaijan University (2019), at the BSP University, Ufa-Russia (2009) for "his contribution in research and international cooperation", and (in 2018) as adjunct Professor at the Ton Duc Thang University – HCMC Vietnam. For the many achievements in research (H-index: Scopus (43, WoS (36), Google Scholar (48)) he has been listed in the 2020 World Ranking of Scientists (Top 2%), ranking 85th (2nd in Italy) for numerical and computational mathematics, and 16th over 335 Italian Mathematicians of this list.

The complete list of publications (1990-2022) can be found at the following web link:

- **Scopus Author ID:** 7004857300
- **WoS Author ID:** I-5051-2013
- https://orcid.org/0000-0002-7504-0424
- https://www.researchgate.net/profile/Carlo_Cattani

Title: *Haar Wavelet Fractional Derivative*

Prof. Dr. David NACCACHE - École Normale Supérieure, Paris, France

Prof. Dr. David NACCACHE was born on February 21, 1967 in Beer Sheva, Israel. David Naccache is a professor at ENS and a member of the ENS' Computer science department. Degree in Engineering, University Paris II, 1990. Master of Science in Theoretical Computer Science, University Paris VI, 1990. Doctor of Philosophy in Cryptology, National Superior School Telecommunications, 1995. Habilitation to Supervise Research, University Paris VII, 2004. He has published over 200 articles in cryptography and information security and is the inventor or co-inventor of 180 patent families. His main research areas are fast implementations, embedded security (smart cards), the detection of security loopholes and public-key cryptography. He is a forensic expert by the Court of Appeal of Paris and formerly by the International Criminal Court (The Hague). David currently serves in the editorial boards of 4 security journals and is an IACR fellow.

Title: *On the security of Internet encryption standards*

Prof. Dr. Anjan BISWAS - Alabama A&M University, USA

Dr. Anjan BISWAS earned his B.Sc (Honors) in Mathematics from St. Xavier's College, Calcutta, India. Subsequently, he completed his M.Sc and M. Phil degrees in Applied Mathematics from the University of Calcutta. Moving further along, he received MA and Ph.D. degrees in Applied Mathematics from the University of New Mexico in Albuquerque, NM, USA. Thereafter, he was a post-doctoral researcher at the University of Colorado, Boulder, CO.

Currently, he works as a faculty member in Mathematics at Alabama A&M University that is in Normal, AL, USA. His current research interest is in Mathematical Photonics. In particular, he focuses on pulse propagation dynamics through optical fibers across trans-oceanic and trans-continental distances. He concentrates on soliton perturbation theory, soliton polarization, quasi—stationary solitons, quasi—particle theory, variational principle as well as semi—inverse variational principle and Lie symmetry analysis that are all applicable to the study of soliton dynamics.

Title: *Quasi-Stationary Optical Gaussons*

Prof. Dr. Ravi P. AGARWAL - Texas A&M University, Kingsville, USA

Dr. Ravi P. AGARWAL was born in Moradabad (India) in 1947. In 1973, he earned his Ph.D. (Mathematics) at the Indian Institute of Technology in Madras, India, one of the highest-ranking universities in India. His major research interests include Numerical Analysis, Differential and Difference Equations, Inequalities, and Fixed-Point Theorems. He has published 45 research monographs and more than 1700 publications in prestigious national and international mathematics journals. Dr. Agarwal worked previously either as a regular faculty or as a visiting professor and scientist in universities in several countries, including India, Germany, Italy, Canada, Australia, USA, Singapore, and Japan. He has been ranked as a Highly Cited Researcher for 14 consecutive years and has also been recognized as one of the "World's Most Influential Scientific Minds" in 2014 and 2015 by Thomas Reuters/Clarivate Analytics.

In 2020 he has been listed among world's Top 2% Scientists by Stanford University. According to Google Scholar, Dr. Agarwal is cited more than 40000 times, and on MathSciNet, his work is cited more than 16000 times by 6400 scientists. Dr. Agarwal is the recipient of several notable honors and awards, including 8 Doctor Honoris Causa and Professor Honoris Causa. He was also nominated as a possible candidate for a Banco Bilbao Vizcaya Argentaria (BBVA) Foundation Frontiers of Knowledge Award, an international award scheme recognizing significant contributions in the areas of scientific research and cultural creation. He has served over 40 Journals in the capacity of an Editor/Honorary Editor, or Associate Editor, and published 20 books as an editor.

Title: *Existence and uniqueness results for a nonlinear integral equation related to infectious disease*

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EQUATIONS ON TIME SCALES:
INTUITIVE PROBLEM-SOLVING THROUGH GAME

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EXTENDED ABSTRACT

Quantum mechanical models are key factors in applications based on quantum effects, with a particular use in the creation and use of a quantum computer. Such models usually belong to the quantum information science – which is a distinct branch related to Computer Science. Nevertheless, the basis for understanding the actual quantum models is acquired through specialized courses in Differential and Partial Differential Equations. Although the necessary scientific background is covered during the first university years, we have identified a learning gap concerning the issue of continuous versus discrete data. Thus, particular models obtained by quantizing continuous data do not always coalesce in order to provide students with the needed knowledge for developing a coherent model for the initial continuous data.

The notion of time scales, along with the corresponding operators: forward, backward, nabla, integral, constitutes a powerful methodology for enabling students to extract continuous patterns from quantized data. Nevertheless, acquiring the theoretical background for mastering these notions is most of the times beyond the required curriculum for passing regular exams. This is the reason why, in the attempt to fill in the gap, we have gone along the same lines as in our previously built adventure games and conceived a new motivating e-challenge – "Time Scales Travel". Our new game requires the player to advance on two equally balanced tracks: the adventure scenario and the mathematical quest. (The former is visible on the levels' game map, while the second is hidden).

In the adventure scenario, the heroin from one of our previous games – "Climbing for clean waters"- returns with a new mission. In this story, she struggles to deal with the threat of an approaching Black Hole. In her quest for a rare mineral needed to build a special time-scales employing smart watch, she passes through different levels until she arrives to the secret lab where a quantum computer can show her the real-time evolution of the menacing Black Hole. The pedagogic element of the game is discreetly hidden behind teleport doors: wrong answers in any of the LevelMaths teleport the player one or two levels backwards in the regular levels of the game. In the math quest, the player has to recognize, through various scientific exercises, notions such as time scales common to different equations, general models corresponding to the same equation expressed on different time scales and particular solutions obtained from the general one. A carefully dosed gradual increase of the difficulty in the LevelMaths' corresponds to the similar challenges that the student will encounter whenever engaging in real-life research. Namely, the player is expected to start from a concrete problem, move on to an abstract model based on it and next accordingly fill in the solution.

CONCLUSIONS

In order to train students to deal with concepts required for working in modern computer laboratories (e.g.: fractional equations, quantum difference equations), the intertwining between the contemporary science-inspired adventure storyline and the math quest makes this serious game a captivating learning experience for students, who can explore the broad field of time scales applications on both mobiles and tablets.

Nevertheless, since the format of challenges is not that of a quiz (with editable content and a randomized order of questions) the game cannot be considered a complete evaluation model for testing the knowledge acquired through either university courses or independent self-paced study.

DYNAMICAL BEHAVIOR OF RANDOM FRACTIONAL INTEGRO-DIFFERENTIAL EQUATION VIA HILFER FRACTIONAL DERIVATIVE

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Key words: *fractional integro-differential equations, Hilfer fractional derivative*

EXTENDED ABSTRACT

This article is committed to studying the random Hilfer fractional integro-differential equation with integral boundary condition. Using Banach and Schauder's fixed point theorems we show that for the aforesaid model the solution exists, is unique and is at least one. Also, Pachpatte's inequality is used in order to provide Hyers–Ulam and Hyers–Ulam–Rassias stabilities results for the mentioned equation. Finally, an example is provided to verify our results.

FRACTIONAL DATA AUGMENTATION FOR IMAGE CLASSIFICATION ON TINY MODELS

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Key words: *fractional calculus; image processing; convolutional neural network; Grünwald-Letnikov derivative.*

EXTENDED ABSTRACT

Image filtering is widely used in different applications of image processing, including the pre-processing and augmentation of images used in Convolutional Neural Networks (CNNs). The use of augmentations such as Gaussian blur, edge detection filters, or high-pass filters enables the training of CNNs on smaller datasets. With the recent emergence of fractional calculus in many areas of science, we wish to revisit the most popular edge detection filters as functions of non-integer order derivatives. Such a fractional edge detection filter allows for another degree of freedom in optimization performance due to the extra free parameter which is the derivative order. This enables the training of smaller CNN models while maintaining the same accuracy as a bigger model. This is because some of the feature extraction workload of the initial layers of the CNN can be reduced by augmenting each image with a fractional-order edge detection filter.To develop the fractional-order edge detection filters we employ the use of the Grünwald-Letnikov derivative:

$$D^s f(x) = \lim_{h \to 0} \frac{1}{h^s} \sum_{m=0}^{\infty} (-1)^m \binom{s}{m} f(x - mh) \qquad (1)$$

In the case when $h = 1$ and the sum is expanded up until its 4th term we obtain the formula:

$$D^s f(x) \approx f(x) - s f(x - 1) - \frac{s(1-s)}{2} f(x - 2) - \frac{s(1-s)(2-s)}{6} f(x - 3) \qquad (2)$$

Equation (2) together with the definition of each edge detection filter considered can be used to obtain the fractional-order edge detection filters.

CONCLUSIONS

We managed to show that by using augmentations consisting of fractional order edge detection filters the performance of a tiny CNN was positively influenced. Also, by comparing the effects of such augmentations on CNNs of different sizes we can say that their effect increases with the decrease in the number of trainable parameters. This allowed us to test the fact that a smaller model using fractional data augmentations can achieve the same classification accuracy on the EuroSAT dataset as a bigger model that doesn't use such augmentations.

$x \rightarrow x^{\alpha}$ TRANSFORMATION AND FRACTIONAL NUMERICAL COMPUTATIONS

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EXTENDED ABSTRACT

In order to numerically solve fractional differential equations, we have to deal with a rising flow of works that show the advantage of using x^{α} instead of x. In this work, we aim to explain this transformation. For this, we focus on the following equation: $D^{q}f(x)=h(x,f(x),g(x))$ with known $g(x)$, and we show that the case $q=\alpha$ gives the best results.

Numerical solutions involve some approximations of the type $f(x)=\Sigma c_n x^{n}$. The transformation $x \rightarrow x^{\alpha}$ leads to the expansion of the function $f(x)=\Sigma c_n x^{\alpha n}$. We will continue to show that this transformation gives us the best results.

To be convincing we consider two methods: fractional-order hybrid of block-pulse functions and Bernoulli polynomials (FOHBPB) [1] and fractional-order Bernoulli wavelets (FOBW) [2]. In the following table we present the results obtained.

X	FOHBPB $q = \alpha = 0.8$	FOHBPB $q=0.8, \alpha = 1$	FOBW $q = \alpha = 0.5$	FOBW $q=0.5, \alpha = 1$
0.1	0	3.3×10^{-17}	1.4×10^{-10}	3.5×10^{-5}
0.3	0	4.2×10^{-17}	1.9×10^{-8}	2.2×10^{-5}
0.5	0	5.6×10^{-17}	1.1×10^{-8}	1.6×10^{-5}
0.7	0	0	7.4×10^{-9}	1.9×10^{-5}
0.9	0	0	7.7×10^{-9}	7.0×10^{-4}

CONCLUSIONS

The conclusions are evident, sometimes with $q=\alpha$ we get the exact results.

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ENHANCING THE ACCURACY OF SOLVING RICCATI FRACTIONAL DIFFERENTIAL EQUATIONS

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EXTENDED ABSTRACT

In this paper, we solve Riccati equations by using the fractional-order hybrid function of block-pulse functions and Bernoulli polynomials(FOHBPB), obtained by making the following substitution:

$$x \rightarrow x^\alpha, \qquad \alpha > 0$$

In the presented development, fractional derivatives are defined in the Caputo sense where the Caputo's fractional derivative of order q has the following definition:

$$(D^q f)(t) = \frac{1}{\Gamma(n-q)} \int_0^t \frac{f^{(n)}(s)}{(t-s)^{q+1-n}} ds, \quad n-1 < q \leq n, \quad n \in \mathbb{N} \quad (1)$$

Using the well-known incomplete beta functions we are enabled to build the Riemann-Liouville fractional integral operator associated with FOHBPB. The incomplete beta function and the Riemann-Liouville fractional integral operator are defined as:

$$B(a, b; z) = \int_0^z t^{a-1}(1-t)^{b-1} dt \qquad (2)$$

$$I^q f(x) = \begin{cases} \frac{1}{\Gamma(q)} \int_0^t \frac{f(s)}{(t-s)^{1-q}} ds, & q > 0, \\ f(t), & q = 0. \end{cases} \qquad (3)$$

Consequently, we make the derivation of the Riemann-Liouville fractional integral operator, noted as I^β, for the fractional-order Bernoulli polynomials, $B^\alpha(x)$. This operator, together with the Newton–Cotes collocation method, allows the reduction of fractional differential equations to a system of algebraic equations, which can be solved by Newton's iterative method. The simplicity of the method recommends it for applications in many scientific areas such as engineering and nature. The accuracy of this method is illustrated by five examples, and there are situations in which we obtain accuracy eleven orders of magnitude higher than the previous known methods when $\alpha = 1$

CONCLUSIONS

In this paper, new functions called fractional-order hybrid functions of block-pulse and Bernoulli polynomials based on Bernoulli polynomials and block functions have been defined. Moreover, we have determined the fractional-order integration and derivative formulas of the fractional-order hybrid functions. The benefits of this method become apparent within the given examples. We considered five examples that clearly indicate the advantages of the current method and illustrate its efficiency. The illustrative examples point towards the conclusion that the presented method is more efficient than the ordinary one, resulting in a great applicability in many scientific areas.

ESTIMATION METHODS FOR SOLUTIONS OF NONLINEAR EQUATIONS

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Key words: *BVP nonlinear problems; estimating of nonlinearities; Green's function; nonlinear evolution PDE.*

EXTENDED ABSTRACT

Nonlinearities are different from linear type by a function, an operator or a system that is nonlinear or is knew only some characteristics of it. Nonlinear is also what is different from what is expected. Even is known that a problem has an exact solution but analytically is not simple to determine sometimes estimations are quite enough depending on the estimation error and what we estimate.

First part of the paper is dedicated to the general framework of the Sturm-Liouville type problem with the operator $L = \frac{d}{dx}\left[p(x)\frac{d}{dx}\right] + \rho(x)$ as part of the regular Sturm-Liouville problem (*RSL*) is considered, using a self-adjoint operator over his domain $D(L) = \{y: I \rightarrow \mathbb{C}, y'' \in L^2(I), y \ satisfies \ the \ boundary \ conditions\}$ and with real eigenvalues and orthogonal eigenfunctions in space $\mathcal{L}^2(I)$ or $\mathcal{L}^2_\mu(I)$, μ the integrant factor depending on L. The solution of RSL will be $u = \sum_{n=1}^{\infty} c_n v_n(x)$ in the complete set $V = \{(v_n)_n, v_n \in D(L)\}$ over $D(L)$.

Among estimation methods we precise homotopy method, tanh-coth method and fractional expansion method. Homotopy method (HAM) is used to estimate the solution of the given BVP problem for the ODE: $f^{(iv)} = A(x)f''' + B(x)f'' + C(x)f' + D(x)f(x) + E(x), x \in [0,1]$, defining the operators $\mathcal{L}$ and N, given by the maximum order of derivation from the equation and by the form of the equation. The zero-order equation associated with the initial equation is expressed:
$$(1-q)\mathcal{L}[\Phi(x,q) - f_0(x)] = hqN[\Phi(x,q)]$$
with h a nonzero parameter, f_0 a first analytical approximation of the function f with conditions known for the BVP (choose for example $f_0 = P_n$ a polynomial function obtained from polynomial approximations of the functions A, B, C, D, E). $\Phi(x,0) = f_0(x)$; $\Phi(x,1) = f(x)$, $x \in [0,1]$ and $\Phi(x,q) = f_0(x) + \sum_1^\infty f_m(x)q^m$; $f_m(x) = \frac{1}{m!}\frac{\partial^m \Phi}{\partial x^m}(x,q)\Big|_{q=0}$. A good choice for h (in relation to the error obtained compared to the initial equation) leads to $f(x) = f_0(x) + \sum_{n=1}^\infty f_m(x)$.

For solitary solutions for NLEEs extended tangent method, exp-function method and G'/G^2 expansion method is discussed looking for solutions of equation $F(u, u_t, u_x, u_{tt}, u_{xx}, \dots) = 0$ where F is polynomial in $u(x,t)$ and its partial derivatives involve nonlinear terms.

CONCLUSIONS

Estimations methods are coming in the paper through an asymptotic study using small parameters, a perturbation study compared with the stability case or a sequential analytical approximant solution that is constructed according with the accuracy established.

FRACTALS IN SOME METRIC SPACES FROM THE PERSPECTIVE OF FRACTIONAL CALCULUS AND METRIC DIGITALIZATION

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Key words: *self-similar fractal (in metric space); sequential (multi) fractal; fractal (fractional) derivative; metric digitalization; (melodic, rhythmic, lexical) coding; enhanced metric spaces; matrix metrics.*

EXTENDED ABSTRACT

The paper generalizes for some metric spaces (mainly compact) the notion of fractal with self-similarity as an invariant compact set (fixed point) for a finite set of contractions that respects a separation condition - - non-loss by overlapping relative to Hausdorff's measure (and included in a true hierarchy) and based on Blaschke's selection theorem [1916] which was originally introduced in Hutchinson [1985] and developed in Falconer [2003]. In addition, relative to the preservation of some contractions in IRn matrices of comparability and equivalence of the usual norms are given.

The notion of fractal is recalled in comparison with that of multifractal commonly used in thermodynamics and extrapolated in music.

From the perspective of fractional calculus, a fractal fractional derivative with three parameters (order, fractal, asymmetry) is proposed, which represents a unification of the respective derivatives with two parameters (order, fractal, respectively order, asymmetry) which are used in recent literature; in addition, the fractal derivative is highlighted as a first step in generalizing the fractal fractional derivative into enhanced metric spaces.

The notion of self-fractal in a complete metric space and in particular of sequential (multi)fractal is used in the study of the properties of some musical pieces.

CONCLUSIONS

The paper proposes for the first time the field of metric digitalization equipped for the beginning with codings (melodic, rhythmic, lexical), enhanced metric spaces and matrix metrics - specific to a sequential fractal (as a particular case) which is then generalized to a sequential multifractal.

An analysis is made for the first time of the (multi) fractal character (melodic, rhythmic and in addition summed up - in terms of n-uplu) of a musical piece without lyrics of great popularity as an alternative to the classical multifractal analysis in statistical frequency, respectively of a famous piece of music with lyrics (by components and with different sums).

CONTROL SYSTEMS IN INTELLIGENT LIGHTING WITH INTEGRATED ALARMS

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Key words*: Fractional PID controller, Integrated alarms, Intelligent lighting system, Rough set, Fuzzy relations.*

EXTENDED ABSTRACT

The idea of smart lighting has emerged over the years in commercial and industrial environments, its primary focus being on improving lighting conditions and through this goal a discovery of the energy saving potential was made. A fuzzy rough set approach is employed in order to classify the boundary line elements of a real system and as such make the method suitable for integration into a smart lighting system.

The approximations of the fuzzy set Y are $\dot{Y} = U\{R < u > |\mu_{R<u>} \leq \mu_Y\}$ (because $A \subseteq B$ only if $\mu_A \leq \mu_B$), $Y^* = U\{R < u > | \exists\, v \in \mathcal{V}^*, (\mu_{R<u>} * \mu_Y)(v) \neq \emptyset\}$. It will be interpreted in the way of the rough membership relation.

We have developed an algorithm to serve in calculating and monitoring the system as a whole. By keeping track of the amount of time a specific light is in use we can estimate the life span of said bulb. The presented algorithm calculates the probability each individual light has of switching on, thus keeping track of the usage metrics throughout any given day. Additionally, the algorithm provides probabilities for each given pair of lightbulbs to be switched on simultaneously which enables us to develop fuzzy relations in the pairs. This enables us to determine the specified fuzzy set Y and calculate its boundary cases in order to interpret the scenario as a rough membership relation.

In this case a regular PID controller would be required to run the scenario, but with upgrading the system this controller would need to be changed accordingly to sustain more functions such as automatic brightness and color temperature adjustment based on outdoor light levels, as well as an integrated alarm system which would necessitate the implementation of a fractional order PID controller (FOPID). However beneficial a fractional-order PID controller may prove, the issue comes with the tuning process as it would require 5 parameters to be tuned thus making the process happen in 5D hyperspace.

CONCLUSIONS

The Building Management System (BMS) gathers a large amount of data from its surrounding environment. To systematically store and retrieve the gathered data, a database would be required. MySQL would be the preferred choice as a database as it is an open source relational database management system. The proposed BMS system stores environmental data into the database over a period of time and performs statistical analysis on the stored data to generate intelligence for predicting future action of the system.

BATTERY USAGE OPTIMIZATION THROUGH EMBEDDED DYNAMIC SYSTEMS

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EXTENDED ABSTRACT

With the improvement of onboard embedded systems and overall processing power battery management systems (BMS) have become the main asset leading the improvement of EVs by allowing full utility of the energy stored in the battery, while also protecting the battery and thereby extending its lifespan. In comparison to other developing technologies relating to this topic, batteries have remained on the rather low end of the spectrum when it comes to growth. This means that EVs still have a relatively high barrier to entry mainly because of the elevated costs of the battery itself.

This has led the rise of model-based SoC estimations as the preferential solution for such problems through their sustained error correction mechanism inside of a closed-loop feedback system. In addition to the efficiency component of the improved estimation that comes with using a fractional order Extended Kalman Filter, the accuracy of the determined parameters could in turn lead to longer preservation of the battery in use and thus result in less of an environmental impact.

The Kalman filter is the optimal linear estimator for linear system models with additive independent white noise in both the transition and the measurement systems. Systems encountered in real word problems invoke non-linearities and therefore an Extended Kalman filter is required in dealing with non-linearity through its prediction and update functions.

The optimality of the path chosen by the robot is to be determined using predicted values of energy consumption acquired through the battery management system. A python type vector is used in order to store the necessary amount of energy for node to node movement. The algorithm will output the path that the robot took in order to get to its destination, the amount of time spent and the amount of energy it took for it to get back to its starting point.
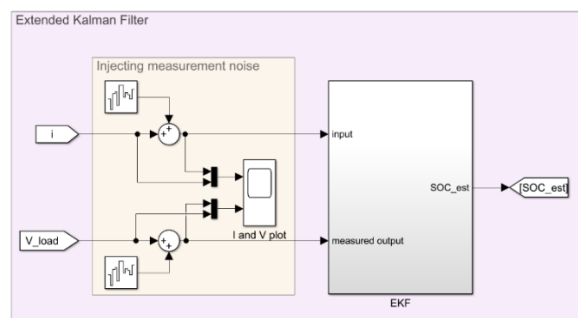
CONCLUSIONS

We have highlighted in this study two examples of implementations of robots in our daily lives which naturally requires a lot more planning and designing of a plethora of different systems but we have chosen to focus on movement and the battery itself because it is the basis on which any project of this sort is built. We strive for the implementation of the Extended Kalman Filter approach directly into the algorithm used for efficiency-based determination of the pathing, thus making the robot capable of choosing a new path in case of an unforeseen event.

ON THE POSITIVITY OF 2D FRACTIONAL LINEAR ROESSER MODEL USING
THE CONFORMABLE DERIVATIVE

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Key words: Positivity, Two-dimensional systems, Roesser model, Fractional calculus, Conformable derivative.

EXTENDED ABSTRACT

In this paper, we investigate the positivity of two-dimensional fractional linear hybrid (continuous discrete-time) systems described by the Roesser model formulated on the conformable fractional derivative calculus. A solution to these class of systems is derived. Necessary and sufficient conditions for the positivity are established. Finally, some illustrative examples are applied to show the applicability and the accuracy of the developed method. The considered model is described by the following equation,

$$\frac{d^\alpha}{dt^\alpha} x^h(t,i) = A_{11} x^h(t,i) + A_{12} x^v(t,i) + B_1 u(t,i)$$
$$x^v(t,i+1) = A_{21} x^h(t,i) + A_{22} x^v(t,i) + B_2 u(t,i) \tag{1}$$

Where $t > 0$ and $i \in Z_+$.
The state matrices $A_{11} \in R^{n_1 \times n_1}$, $A_{12} \in R^{n_1 \times n_2}$, $A_{21} \in R^{n_2 \times n_1}$, $A_{22} \in R^{n_2 \times n_2}$, $B_1 \in R^{n_1 \times m}$, $B_2 \in R^{n_2 \times m}$, $u(t,i)$ is the input vector and the fractional differential equation is defined the following equation

$$\frac{d^\alpha}{dt^\alpha} x(t) = lim_{\epsilon \to 0} \frac{x(t+\epsilon^{1-\alpha})-x(t)}{\epsilon} \tag{2}$$

CONCLUSIONS

In this paper a new class of two-dimensional fractional linear systems described by the Roesser model and formulated by the conformable derivative is introduced. The solution formula of the considered system with the initial conditions has been derived. Necessary and sufficient positivity conditions have been established. The considerations can be extended to the descriptor system (singular and non-singular cases). Moreover, the asymptotic stability of the introduced model should be derived.

DUFFING-VAN DER POL OSCILLATOR UNDER FRACTIONAL-ORDER DELAYED FEEDBACK AND EXTERNAL FORCES

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Key words: *Duffing-van der Pol equation; averaging method; homotopy analysis.*

EXTENDED ABSTRACT

In this paper, the dynamics of a Duffing-Van der Pol oscillator under fractional-order delayed feedback control and external forces is investigated both analytically and numerically. The fractional derivative is described in the Caputo sense. The primary resonance of the fractional oscillator under consideration is studied using the averaging method and the homotopy analysis principle. Then, the results of approximate analytical solution and numerical integration are compared by numerical simulation. The effects of time-delayed feedbacks and fractional derivative on the steady state response are also studied.

CONCLUSIONS

The dynamical behaviour of a Duffing-Van der Pol oscillator under fractional-order delayed feedback control and external forces is discussed. By the slow-flow equations, the stability condition of the steady-state solution is obtained. The periodic bifurcations are examined using the fractional order, feedback gain and time delay as continuous parameters. It could be concluded that the fractional order, the feedback gains and time delays have important effects on the amplitude of the periodic solution.

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MULTILEVEL INTERPOLATION OF PERIODIC FUNCTIONS WITH GAUSSIANS

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Key words: *multilevel; interpolation; periodic functions; gaussians; radial basis functions.*

EXTENDED ABSTRACT

In the approximation area, the high-dimensional challenges are always hard due to the curse of dimensionality. In this paper, we provide the error analysis of the algorithm for approximating the cosine frequencies, and hence even functions. We can also prove the same result for odd functions, it is essentially the same. We extend these estimates to the multilevel context, for interpolation. We look at periodic functions in one dimension, i.e., those defined on the unit circle T. Let $C_{1}(T)$ denote the linear space of 1-periodic continuous functions, and $0 \le x_{1} < \ldots < x_{n} < 1$ denote our data points. In [1], Kushpel developed the theory of sk-splines, where, for a fixed kernel function K: $T \rightarrow R$, we approximate $f \in C_{1}(T)$ with a function. The work in this paper has grown out of the desire to break the curse of dimensionality using sparse grids, and the Gaussian as a kernel. We analyse univariate multilevel interpolation. We consider the periodic case since this allows one to use the tools of Fourier analysis. Even so, the analysis is complicated. As it turns out, interpolation is easier to analyse since the convergence is much faster. The numerical experiments for interpolation show that the error for approximating the frequencies goes to 10^{-13} accuracy. Overall, our multilevel interpolation algorithm has a faster convergence so we can give error estimates to a greater precision.

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EXISTENCE THEORY AND ULAMS STABILITIES FOR SWITCHED COUPLED SYSTEM OF IMPLICIT IMPULSIVE FRACTIONAL ORDER LANGEVIN EQUATIONS

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EXTENDED ABSTRACT

In this manuscript, switched coupled system of nonlinear implicit impulsive Langevin equations with two Hilfer fractional derivatives is considered. Using the techniques of nonlinear functional analysis, we establish appropriate conditions and results to discuss existence, uniqueness and Ulam's type stability results of our proposed model, with the help of Banach's fixed point theorem. An example is provided at the end to illustrate our results.

CONCLUSIONS

In this article, we consider switched coupled system of implicit impulsive LEs with four HFDs. Some conditions are made to beat the hurdles to investigate the existence, uniqueness and to discuss different types of UH stability of our considered model, using Banach's fixed point theorem. We give an example which supports our main result.

INFLUENCE OF DISCRETIZATION STEP ON POSITIVITY OF CONFORMABLE FRACTIONAL LINEAR SYSTEMS

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Key words: *Fractional linear systems; Conformable derivative; Positivity; Discretization; Discrete systems..*

EXTENDED ABSTRACT

The concepts of the conformable derivatives and their properties are considered in this work. A continuous time linear systems and their positivity are studied. A method is proposed in this paper to study the relation between the value of discretization step and positivity for a certain class of continuous time linear conformable fractional systems i.e., under which conditions the one-dimensional discrete-time linear system obtained by discretization from the one-dimensional continuous time linear systems will be also positive if the one-dimensional continuous time linear system is positive. An illustrative examples are finally proposed. The considered model is described by the following equation,

$$\frac{d^{\alpha}}{dt^{\alpha}}x(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t) \tag{1}$$

Where $t > 0$.
The state matrices $A \in R^{n \times n}$, $B \in R^{n \times m}$, $C \in R^{q \times n}$, $D \in R^{q \times m}$, $u(t)$ is the input vector and $y(t)$ *is the output vector.* The fractional differential equation is defined the following equation

$$\frac{d^{\alpha}}{dt^{\alpha}}x(t) = lim_{\epsilon \to 0}\frac{x(t+\varepsilon^{1-\alpha})-x(t)}{\varepsilon} \tag{2}$$

CONCLUSIONS

In this paper, the study of the influence of the value of sampling step on positivity of one-dimensional discrete-time linear system obtained by discretization from one-dimensional conformable fractional linear system is investigated. Necessary and sufficient conditions are then proposed and two numerical examples that illustrate the applicability of the proposed results.

METHODS OF MULTIFRACTAL EXTRAPOLATION IN CASE OF MUSICAL PIECES WITH LYRICS

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Key words: *Multifractals, Sequential Multifractal, Metrical Digitalization, Music with Lyrics, Python;*

EXTENDED ABSTRACT

This research conducts a study on establishing a correlation between theory of music and mathematics formalisms, more precisely, it uses multifractals for the extrapolation of musical pieces for its further structural and graphical representation, which may help in finding a different approach in how we see musical pieces. These structures of musical multifractals may have a much deeper meaning than simply sounding repetitive which corresponds with the classical method while the metrical digitalization method offers a vectorial perspective with the help of the lyrical component.

As an example, this research uses the musical interpretation of the poem "Pe lângă plopii fără soţ", written by Romanian national poet Mihai Eminescu. Done in a classical manner, this is a complete study of a musical piece, including both the musical component as well as the lyrical aspect, using multifractals in the thermodynamical sense.

With application of the most modern computer science notions data is coded and represented intuitively thus making its understanding reachable for anyone. The research makes use of matrices and metric spaces in order to represent the melody, the rhythm and the lyrics, combining them so as to obtain a full picture.

CONCLUSIONS

This paper proposes a new way on analysing musical pieces, both with and without lyrics. It explores a field of linking music with mathematics by encoding musical pieces, paving a path for finding new properties that music might have. Using the methods applied in the research one can obtain new results to study. They may be used, for example in creation of a new format of sound files which can lead to a more efficient computer storage usage.

STARLIKE FUNCTIONS BASED ON RUSCHEWEYH $Q-$ DIFFERENTIAL OPERATOR DEFINED IN JANOWSKI DOMAIN

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EXTENDED ABSTRACT

In this paper, we make use of the concept of q-calculus
in the theory of univalent functions, to obtain the bounds for certain coefficient functional problems of starlike functions also some inheriting results like the Fekete-Szegő functional. A similar results have been done for the function $f^{-1}.$ Further, coefficient estimates, distortion bounds, radius problems, a number of sufficient conditions and results related to partial sums are derived.

CONCLUSIONS

In this paper, for this newly defined functions class, we have studied a number of well-known results like the Fekete-Szegő inequalities, the necessary and sufficient
conditions, the growth and distortions bounds, the radii of close-to-convexity and starlikeness and patial sums type results.
Furthermore, we believe that, this study will motivate a number of researchers to extend this idea for meromorphic functions and harmonic functions. One may also apply this idea in ordered to shell-like domain and petal shaped domain instead of the Janowski domain.

HIGHLY DISPERSIVE OPTICAL SOLITON PERTURBATION WITH COMPLEX—GINZBURG LANDAU MODEL BY SEMI—INVERSE VARIATION

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EXTENDED ABSTRACT

The dynamics of perturbed highly dispersive optical solitons is studied in this work. The governing model is the complex Ginzburg—Landau equation with six dispersion terms. The perturbation effects appear with maximum allowable intensity or full nonlinearity. Three forms of self—phase modulation are considered. They stem from Kerr effect, parabolic law and finally the polynomial form. The semi—inverse variational principle is implemented to recover bright 1—soliton solutions to the model which is otherwise non—integrable with any of the known integration schemes. The applied principle retrieves analytical, but not exact, bright 1—soliton solutions to the model. The parameter constraints, that guarantee the existence of such solitons, are also identified, and presented.

STATISTICAL ESTIMATION TECHNIQUES USED IN CRYPTOGRAPHIC EVALUATION

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Key words: statistical estimation; random number generators; data analysis and information security.

EXTENDED ABSTRACT

The evaluation of cryptographic algorithms and protocols is based on the following:
- simplicity and transparency of the description;
- flexibility of implementation on various categories of processors;
- robustness to hardware, firmware and software implementations;
- processing speed and memory requirements;
- resistance to cryptanalytic attacks.

The latter also cover statistical testing and evaluation of the minimum entropy of random generators used in cryptographic applications.

One of the standards proposed to be used in evaluation of minimum entropy are SP-800-90 composed by: SP 800-90A Rev. 1, Recommendation for Random Number Generation Using Deterministic Random Bit Generators, and SP 800-90B, Recommendation for the Entropy Sources Used for Random Bit Generation.

Recall that the min-entropy of an independent discrete random variable X that takes values from the set $A=\{x_1,\ldots,x_k\}$ with probability $Pr(X=x_i)=p_i$ for every $i=1,\ldots,k$ is defined by:

$$H_\infty = - \min_{1\leq i\leq k}(\log_2 p_i) = - \log_2 \max_{1\leq i\leq k}(p_i)$$

Min-entropy is a limiting case ($\alpha \rightarrow \infty$) of Rényi entropy:

$$H_\alpha = \frac{1}{1-\alpha} log \sum_{i=1}^{k} p_i^\alpha$$

Others limiting cases of Rényi entropy are Shannon entropy H_1 ($\alpha \rightarrow 1$), collision entropy H_2 ($\alpha=2$) and max-entropy H_0.

The scope of this work is to extend the algorithms proposed in NIST 800-90A and NIST 800-90B, used for entropy estimation (min entropy), to Rényi entropy estimation.

CONCLUSIONS

The extensions of the algorithms proposed in NIST 800-90 will allow us to find the estimation of min entropy, Shannon entropy, and collision entropy like a limiting case of the estimation of the Rényi entropy.

GAMES THEORY INVOLVEMENT IN THE SOCIAL ROBOTS FACIAL EXPRESSIONS

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EXTENDED ABSTRACT

This paper presents a method based on games theory and coordination graphs with the aim of improving the social robot facial expressions to perform human-like motions. The method comes as an alternative to the traditional neural networks or fuzzy logic methodologies. The difficulty of such robots is raised by the elevated number of servomotors/Degrees of Freedom (up to 35 Degrees of Freedom (DOFs)) used to control the entire robotic system, but also to translate the motions of more than 40 face muscles to get the proper facial expressions using the facial action coding system (FACS). In order to achieve this goal, the facial expressions of a social robot (Android) were decomposed into different sections as sum of action units, and were obtained multiple coordination graphs of reduced dimensions which resumed the problem to cooperative multi-agent decision making. The global payoff function of each section of the face will be a sum of local terms, meaning the payoff function of each player and the messages between neighbouring players respectively. Four different emotions (happy, sad, fear and anger) were described using coordination graphs. The games are described by a matrix of three players or five respectively, and the dominance and equilibrium states are studied using Gambit software. The real-time decision making will be applied to a centralized max-plus algorithm. The method of controlling servos of social robots using games theory is an innovative idea and a key factor for such applications.

CONCLUSIONS

In this paper we have proposed a method based on games theory which is an innovative idea for Androids designed with facial expressions. The Android face is segmented into several action units, then coordination graphs are proposed to obtain facial movements/emotions and integrate all servomotors of the Android in the entire game. As many servomotors are involved in the coordination graphs, as much the facial expressions are close to the human-like motions. The facial muscles are well mimicked if the face has as much possible degrees of freedom. The method was implemented only on cooperative games, such as soccer games or drones coordinated flying. Here, the problem is more complex because the servomotors are interconnected and should converge to different but complex expressions. It is not only a mechanical movement that servomotors should execute; it should be obtained a smoothness motion, not dangerous as in surgical robotics, but to obtain a human-like face movement. The equilibrium and dominance of every strategic game with multiple players were obtained using Gambit software. Future work will include the Max-Plus Algorithm implementation for four basic facial expressions (emotions), then will be extended to a various range of facial expressions.

THEORETICAL FOUNDATIONS FOR LIFELONG CYBER-PHYSICAL SYSTEMS

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Key words: *Lifelong cyber-physical systems; compact mathematical modelling; fractional dynamical networks; anomaly detection.*

EXTENDED ABSTRACT

Lifelong cyber-physical systems (CPSs) are expected to provide a paradigm shift in future Internet-of-Things infrastructure by providing compact perception, efficient reasoning and decision-making capabilities in a wide areas of application areas irrespective of software and hardware changes and malfunctions. Towards this end, the conception, design and implementation of lifelong CPSs need to take into account the mathematical characteristics of both the cyber and physical components – complex spatio-temporal multifractal properties that require new mathematical operators and dynamical equations learned from data. We describe a set of mathematical strategies for constructing compact yet accurate mathematical models that endow the future CPS with compact perception and robust decision-making capabilities [1]. Simply speaking the non-Markovian and multifractal properties of cyber and physical processes can be captured through a set of space and time fractional order operators providing a compact dynamic representation of complex CPS – least number of model parameters. This mathematical strategy not only offers new controller design strategies, but also the opportunity to develop new machine learning and artificial intelligence tools for dealing with the evolving requirements of lifelong CPS. We discuss the benefits the multiwavelets neural operator architectures bring on learning partial differential equations from real time data [2]. To demonstrate the impact of these mathematical approaches, we describe how we can model noisy phasor measurement units (PMUs) data and proactively detect anomalous behaviour [3]. In addition, we discuss and exemplify the benefits of these new mathematical strategies to learn dynamical models and forecast not only anomalous events in power grids – like the voltage blackout, but also COVID19 epidemic evolution [4]. We conclude by summarizing a few major challenges concerning the combination of efficient learning with real-time decision-making.

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STEGANOGRAPHY APPLICATION FOR ANDROID

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EXTENDED ABSTRACT

In order to be able to offer users the opportunity to apply an invisible watermark and then be able to prove that the picture is indeed signed by users, More steps had to be taken to develop the aplication, we created a database in which we store the user's email address, password and a watermark automatically generated by the application based on his email address thus guaranteeing the uniqueness of the watermarks and the possibility of associating each watermark with the user who owns it. In order to apply an invisible watermark we used an algorithm based on LSB (least significant bit) in the spatial domain in which each 8-bit pixel's least significant bit is overwritten with a bit from the watermark.

Given the extraordinarily high channel capacity of using the entire cover for transmission in this method, a smaller object may be embedded multiple times.

CONCLUSIONS

The proposed application offers the possibility to sign pictures to prove the right to property by applying watermarks but without damaging their quality.

AUTOMATIC IDENTIFICATION OF SIMILARITIES BETWEEN FUNCTIONS OF MALWARE FILES

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EXTENDED ABSTRACT

This paper presents a comparison of existing solutions of automatic identification of similarities between malware files and proposes a way of detecting if an unknown file is either malware or not by using machine learning.
In the current context, hundreds of millions of new variants of malware are found every year, the most current methods of detecting malware, such as YARA rules, end up being obsolete. This has led to the need to develop detection programs that use machine learning, as they can learn to detect and classify new malware.

Methods and Results

Due to the lack of public datasets of malware files, a proper dataset was created for this work. This dataset consists of 5 families of malware files, each family consisting of 15 files, and another 75 benign files taken from a newly installed Windows operating system.
Given the small size of the dataset, in order to find the best results, we decided to divide the initial dataset into 3 individual modes: one dataset 80-20, another 70-30, and the last one 60-40, where the first value represents what percentage of the dataset content will represent the training set, and the second value what percentage will represent the test set.
The existing solutions that will be compared are represented by TF-IDF with cosine similarity and Random Forest Classifier, resulting 6 instances (3 datasets * 2 models). We apply TF-IDF over the training and test datasets and then we calculate the cosine similarity for each entry in the resulting test dataset with all the entries in the final train dataset. The Random Forest Classifier is trained on the train dataset resulting from parsing the original train dataset and testing it with a test dataset resulting from the same parsing only this time on the test dataset.
In order to lower the numbers of False Positive and False Negative, we added in the test datasets another 3 malware families, each consisting of 15 files. We analyzed how sure each instance is about it's predictions and set up thresholds that tell the instance whether to trust the prediction it made or classify the file as "Unknown File".
The paper will present the statistics and the results obtained following the individual application of the 3 types of datasets on the two models (TF-IDF with cosine similarity and Random Forest Classifier).
The proposed detection model that will detect if a file contains malware or is benign will result from the statistics mentioned above.

CONCLUSIONS

It is important to note that a detection program that uses machine learning will greatly facilitate the detection and classification of new versions of malware. This is necessary given the large number of new malware files that appear every year.

AUTOMATED PRODUCTS SORTING SYSTEM FROM E-SHOPS

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EXTENDED ABSTRACT

Nowadays, it does not exist an e-commerce platform that does not include a recommendation system for their products. This practice encourages customers to spend more money by offering products that correspond to their needs. Living in the 21st century where people are busier than ever it is of utmost importance that time spent choosing from different products is reduced. In order to achieve this recommendation systems are constantly evolving. This paper aims to present the implementation of such a recommendation system. As far as I know, the biggest fashion e-commerce platforms available at the moment do not allow the user to upload their own photo with the purpose of purchasing similar products. In this manner, I chose a solution based on visual similarity and semantic similarity, but the uniqueness relies on the matter presented above. Regarding this option, the system detects clothing objects presented in the image and crops it in other images containing detected fashion objects. These operations are made using YOLOv5 algorithm. Then, each image is taken and the recommendation system analyzes it and tries to recommend products that are similar in some proportion with the product received as input. For visual recommendation, the Artificial Intelligence technique I used is named Transfer Learning. The idea behind this technique is to extract embeddings from some representations and use them as input to another model. For extracting these embeddings of the images found in the dataset I used ResNet or Residual Network which is one of the famous deep learning models for image recognition. I saved these features into a pickle file. In order to find the most similar images to the one chosen by the user, I extract the embeddings of the chosen image and then compute the Euclidean Distance via Nearest Neighbours between the features of this image and those saved in the pickle file. Regarding the semantic search, I used Natural Language Processing techniques names TF-IDF and LSI. TF-IDF is the abbreviation of Term Frequency-Inverse Document Frequency and it represents an NLP model which determines the most important words from the corpus created before using the description of each image present in dataset. As soon as this is ready, TF-IDF model is passed as input to LSI model. LSI examines a collection of documents to see which documents contain some of those same words. LSI considers documents that have many words in common to be semantically close and ones with less words in common to be less close. LSI does not require an exact match to return useful results. Also, for comparison, I implemented a module for product search, but based on the BERT module, a state-of-the-art open-source machine learning framework for Natural Language Processing.

CONCLUSIONS

We further confine to the opinion expressed by Peter Olver in his celebrated work, which says that 1-jet spaces and their duals are appropriate fundamental ambient mathematical spaces used to model classical and quantum field theories. In such a physical and geometrical context, followed papers which are devoted to developing the *time-dependent covariant Hamilton geometry on dual 1-jet spaces* (in the sense of d−tensors, time-dependent semisprays of momenta, nonlinear connections, N-linear connections, d−torsions and d−curvatures), which is a natural dual jet extension of the Hamilton geometry on the cotangent bundle.

COMPARISON BETWEEN THE WAVELET TRANSFORMS USED FOR PARAMETERS EXTRACTION IN DIGITAL FINGERPRINTS

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EXTENDED ABSTRACT

The influence of the wavelet transforms on the parameter extraction for fingerprints classification is investigated. The analyzed fingerprint images belong to the FVC2004 database, which is the property of the University of Bologna, Italy [*]. 104 real images belonging to BD2 dataset were acquired using an Optical sensor "U.are.U 4000" and have 328×364 pixels size. 104 synthetic generated images belonging to BD4 dataset have 288×384 pixels size.

The classification is performed based on the wavelet packet transform and dimension reduction. Four wavelet transform families with a total of 16 wavelets were used as follows: Coiflets (coifN, where N = 4), Symlets (symN, where N = 4), Fejer-Korovkin (fkN, where N = 4) and Biorthogonal (Bior, where N = 3 and 5). These wavelet transforms are used for comparison. For classification purpose, the energy and entropy statistical parameters are associated with the wavelet transformations and a 32 features vector size is built. The principal component analysis (PCA) is used for dimensional reduction. Then, the obtained characteristics vector is delivered to a k-Nearest Neighbor (k-NN) classifier. The success rate results in the classification for each dataset and features associated to the wavelet transform packet is analyzed.

According to the data provided by the eigenvalues, those variables that correlate to the first two components are considered to be the most important in explaining the variability of the initial data (77.11% for BD2, 70.13% for BD4), followed by those associated with PC3, PC4 and PC5. Thus, the first five PCs are preserved as they cumulatively carried out 93.611% of the information and respectively, 90.929% of the total variance of the point.

CONCLUSIONS

The fingerprints classification was achieved by analyzing the features of waveforms with the energy and entropy statistical parameters associated to the wavelet transform packet along with dimension reduction, using the principal component analysis (PCA). The fkN filters provides the best results as the frequency response gets more accurate results even when compared to the other filters utilized.

*FVC2004: Third Fingerprint Verification Competition. Available online: http://bias.csr.unibo.it/fvc2004/databases.asp (accessed on 10 February 2022).

MATHEMATICAL SPATIAL DOMAIN-BASED APPROACH FOR QUALITY ASSESSMENT OF MULTI-EXPOSED BREAST IMAGES

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Key words: *multi-exposed breast images; contrast variation; gradient maps; radial basis function neural network; quality regression.*

EXTENDED ABSTRACT

Spatial domain is manipulating the image content in pixels' space to enhance it for a given application.
The image contrast is measured by the range of gray levels intensity. Images having a higher contrast level show a greater grayscale variation than those of lower contrast. An insufficient contrast can result in underexposure of image details and few intensity levels are available to illustrate the subtle differences of intensity that may occur in the breast image. The goal of contrast manipulation techniques, by means of the over-exposure and the under-exposure, is to accentuate certain image features but with detail preservation. The structure features, namely, the contrast variation and gradient maps are investigated to quantify the influence of abnormal exposure. The gradient maps are used to describe the structure loss phenomenon. Moreover, the structural variation is considered by refining the gradient maps based on the local mean subtraction and standard deviation normalization.
The similarities of raw images' gradient maps and manipulated images' gradient maps are investigated to quantify the influence of abnormal exposure. The radial basis function neural network (RBFNN) is employed for quality regression problem investigation.

CONCLUSIONS

This study aims to analyse contrast manipulation techniques, using over-exposure and under-exposure, to accentuate certain image features but with detail preserving.
To determine the visual quality of a multi-exposure fused (MEF) image, the investigation searches whether the structural information for over-exposed and under-exposed regions is damaged or not.
Two sets of features feed a radial basis function neural network (RBFNN) for malignant vs benign classification.
For the first set of features, extracted with relation which define the gradient combined with the exposure, an accuracy of 78.6% was obtained.
For the second set of features, extracted with relation which define the gradient is combined with the exposure but also with the mean normalized contrast coefficients, an accuracy of 82.4% was achieved which shows that this type of images better classify malignant vs benign breast tumors.

INTERLOCUTOR PROFILE DETECTION SYSTEM

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Key words: *Gpt-3, Profile detection, BERT, chatbot*

1. EXTENDED ABSTRACT

With the exponential growth of artificial intelligence, new techniques have been developed to create conversational agents, which has led to their evolution in understanding natural language and their spread into many areas such as e-commerce, real estate, healthcare or personal use as virtual assistants. Although they have proven their usefulness, however, there are cases where they can be used for purposes that cannot be considered ethical, such as spreading fake news or trying to manipulate people.

This work aims to create an automatic system for detecting the profile of the interlocutor, human or virtual, from a real-time interaction carried out through the Telegram application. In other words, it is proposed to create a Turing Test. For this purpose, two types of detection have been chosen: passive and active. Passive detection extracts temporal data, such as response time, and spatial data, such as the size of a message in words, while passive detection aims at extracting features from each received message through artificial intelligence modules. These features are spam detection, linguistic acceptance of a message, dominant sentiment detection and bias detection of a message. To automate the whole feature extraction process, a bot has been created using the innovative GPT-3 technology, which will respond to incoming messages from interlocutors on behalf of the system user.

CONCLUSIONS

This work presents all the steps that have been considered for the creation of the solution, from the presentation of some theoretical notions, to the system design and implementation of the solution with all the aspects that have been taken into account.

ANALYSIS OF A DDE MODEL OF EVOLUTION IN CHIKUNGUNYA INCORPORATING THE ACTION OF THE IMMUNE SYSTEM

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Key words: *Delay Differential Equation, Chikungunya virus, Ribavirin, Stability Analysis, Critical Case.*

EXTENDED ABSTRACT

Introduction: Chikungunya disease (CHIKD), caused by Chikungunya virus (CHIKV), is becoming a major threat to public health. The past few decades have seen the re-emergence of this viral infection as evidenced by epidemics in Africa, Asia and Europe and in recent years, the Americas. Using in vitro experiments, the possibility of detecting Chikungunya virus in the monocytes of severely infected patients was confirmed. Therefore, monocytes could be infected and virus growth could be sustained. The prevalence of joint inflammatory diseases in chronic CHIKV infected patients was shown to be linked with the persistent of CHIKV RNA in the perivascular synovial macrophages.

Results: We introduce a system of delay differential equations, modelling Chikungunya evolution, considering the action of the immune system and under treatment with Ribavirin. This system consists of 12 equations with 10 delays representing the stem-like healthy cell population, uninfected and infected monocytes, the pharmacokinetic of the antiviral Ribavirin in the tissue, viral burden and the immune system.
This model has 2 types of equilibrium points, E_1, representing the last stage of the disease and E_2 that can be viewed as a healthy state. A critical case appears in the characteristic equations of E_1 and E_2. The study of stability of equilibria of a system of DDEs is often based on the study of the eigenvalues of the linearized system, through the theorem of linear approximation. A zero eigenvalue represents an especially complex case, which is called a critical case, since that theorem cannot be applied. This type of systems, for ordinary differential equations, were studied by Malkin, where a theorem for the stability in this critical case is proved. We extended this theorem to the case of DDE systems. The main tool is the use of a complete Lyapunov-Krasovskii functional.
To apply this theorem, we perform first a translation to zero. Then, after some assumptions and calculations, we conclude that the general theorem on the critical case works for the system and the stability depends on the study of the transcendental terms in the characteristic equations of E_1 and E_2.
The trajectories starting in a neighbourhood of equilibrium E_1, representing the most aggravated phase of the disease, are plotted. One can easily notice that in this case the variables representing the healthy monocyte population and precursors are unstable while the variables representing the Chikungunya virus population and the infected monocytes population are stable. From a medical point of view, this might translate into the recovery of the patient, since the Chikungunya virus and the infected monocytes are vanishing while the healthy monocytes are recovering. The trajectories starting in a neighbourhood of the equilibrium E_2, representing the disease-free state, are also plotted. Here the stability of the variables representing Chikungunya virus population and the infected monocytes population together with stability of the variables representing healthy monocyte population and precursors fits the expected outcome.

A STUDY OF MATHEMATICAL MODELS USED IN THE CEREBRAL HEMODYNAMIC CIRCULATION

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Key words: mathematical modeling; linear models; parameter estimation; confidence; Windkessel model.

EXTENDED ABSTRACT

The purpose of this study was twofold: 1) apply different forms of Windkessel models to the human cerebral circulation, and 2) compare the experimental data and the best fit of parametric lumped models. To address the study aim, we tested 24 healthy adults between the ages of 22 and 80 years old. The arterial blood pressure (BP) and cerebral blood flow velocity (CBFV) were used as the input and output signals respectively, during the hemodynamic parameter estimation. The arterial pressure waveform was measured from the common carotid artery while simultaneously recording CBFV from the middle cerebral artery using a transcranial Doppler probe. The data were collected with a sampling frequency of 1KHz and model parameter estimates were obtained after post-processing. Different linear models were studied in the parameter estimation of the Windkessel model using the black box identification based on linear models: the autoregressive model (ARX), the autoregressive moving average model (ARMAX), the output error (OE) and the Box Jenkins (BJ) method. The transfer function for each Windkessel model, with two, three and four elements respectively, were associated with a corresponding linear model, having the same degree (equations of first order or second order). Knowing the order of the transfer function, based on the input and output measurements was obtained a time domain transfer function with corresponding poles and zeros, converted then in the frequency domain (s-domain), a transfer function identified with the Windkessel transfer functions. The results showed that a Box Jenkins linear model provides good confidence values for the measured and simulated curves, and that a Windkessel model with four elements is closest to expressing cerebral hemodynamic circulation.

CONCLUSIONS

We studied Windkessel models of human cerebral circulation with the increasing number of elements. Based on an assumption of the linear systems, we estimated the Windkessel parameters using the Autoregressive model (ARX), Autoregressive Moving Average model (ARMAX) model, the Output Error (OE) model and Box Jenkins (BJ) model respectively. The hemodynamic data were collected from a group of young adults, and into a next step from old subjects. The data sets allowed us to conclude that the circulatory hemodynamic can be modeled and estimated using a 4 element model for both young and old subjects. The identified parameters of the model may provide physiological interpretation of effects of aging of the cerebral circulation.

AN ORIGINAL VERSION OF THE SAMPLE ENTROPY ALGORITHM FOR BINARY SEQUENCES

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Key words: *criptography, binary sequences, sample entropy, approximate entropy, algorithm.*

EXTENDED ABSTRACT

Cryptographic techniques play a key role in ensuring data security, and in these techniques, the use of random number sequences is the core of any cryptographic system for choosing the initialization vector needed to produce keys.
In this paper we propose and analyze an original variant of the Sample Entropy algorithm for binary sequences and we compare this original version with the Approximate Entropy algorithm, the latter being included in the NIST series of tests. Entropy is defined, in this context, as a measure of the degree of randomness for a binary sequence (Shannon's Entropy). In the latter part of the paper there is a comparative analysis of the two algorithms: Sample Entropy and Approximate Entropy for binary sequences, obtained using a random number generator as well as non-random sequences to discuss and compare the Approximate Entropy and the Sample Entropy output functions (as measures of entropy).

CONCLUSIONS

The output function of the proposed algorithm, SampEn, has always a greater value than the output function of the NIST test – ApEn, due to the fact that in calculating SampEn we exclude the comparison of the block with itself. Therefore, the degree of randomness of a binary sequence (as a measure of entropy) is no longer penalized by the comparison of a block with itself. The comparisons made for 1000 binary sequences (each of 1000 bits) confirm that SampEn is a better measure of randomness.

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MINIMAX RATES OF CONVERGENCE FOR STATISTICAL INVERSE PROBLEMS BASED ON DISCRETELY SAMPLED FUNCTIONAL DATA

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Key words: *optimality, minimax rates, functional data, linear inverse problems, regularization methods*

EXTENDED ABSTRACT

We focus in this paper on the optimal parameter estimation from noisy independent realizations of a stochastic process that is indirectly observed (see [1,2,3]). First, we study minimax rates of convergence for a class of linear inverse problems with correlated noise, general source conditions and various degrees of ill-posedness for entirely observed pathes. In the framework of the projected data, two settings are presented separately: the case of the common design (when the set of test functions are all identical) and the case of the independent design. The latter reduces to the former under the specific conditions of the common design and has an interpretation in the view of the sampling properties via s-numbers (see [4,5]). The phase transition can be observed also here as it is usual for the functional data analysis.

CONCLUSIONS

We compute the thresholds that separate the sparse and the dense data set settings for different smoothness conditions, compare the optimal rates of convergence in different scenarios with the rates corresponding to the statistical inverse problems and observe that the price to pay for the data correlation proves to be high (see [6]).

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STUDY ON TIME SCALE STABILITY OF NEUTRAL NONLINEAR DYNAMIC EQUATION WITH UNBOUNDED DELAY

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EXTENDED ABSTRACT

Let $\mathbb{T}$ be a time scale which is unbounded above and below and such that $0 \in \mathbb{T}$. Let $id - r : \mathbb{T} \to \mathbb{T}$ be such that $(id - r)(\mathbb{T})$ is a time scale. we use the contraction mapping theorem to obtain stability results about the zero solution for the following neutral nonlinear dynamic equations with unbounded delay

$$x^{\triangle}(t) = -a(t) x^{\sigma}(t) + b(t) G\left(x^2(t), x^2(t - r(t))\right)$$
$$+ c(t) \left(x^2\right)^{\tilde{\triangle}}(t - r(t)), \ t \in \mathbb{T},$$

We provide interesting examples to illustrate our claims.

CONCLUSIONS

We have studied dynamic nonlinear equations with functional delay on a time scale and have obtained some interesting results concerning the existence of periodic solutions and this work is a continuation. Here, we focus on two neutral nonlinear dynamic equations which, for our delight, have not been yet studied by mean of fixed point technic on time scales.

There is no doubt that the Liapunov method have been used successfully to investigate stability properties of wide variety of ordinary, functional and partial equations. Nevertheless, the application of this method to problem of stability in differential equations with delay has encountered serious difficulties if the delay is unbounded or if the equation has unbounded term. It has been noticed that some of these difficulties vanish by using the fixed point technic. Other advantages of fixed point theory over Liapunov's method is that the conditions of the former are average while those of the latter are pointwise.

DYNAMICAL SYSTEMS GENERATED BY KASNER QUADRILATERALS WITH COMPLEX PARAMETER

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Key words: *dynamic geometry; Kasner quadrilaterals; complex coordinates; nested quadrilaterals.*

1. EXTENDED ABSTRACT

For a real number α and an initial quadrilateral $A_0B_0C_0D_0$, one can construct the quadrilateral $A_1B_1C_1D_1$ such that A_1, B_1, C_1 and D_1 divide the segments $[A_0B_0]$, $[B_0C_0]$, $[C_0D_0]$ and $[D_0A_0]$ respectively, in the ratio $1- \alpha : \alpha$. Continuing this process one obtains a sequence $A_nB_nC_nD_n$, $n \geq 0$, whose terms are called Kasner quadrilaterals (after E. Kasner (1878-1955) who studied in detail the case of triangles), or nested quadrilaterals in other references.

Related examples of iterative processes inspired by simple geometrical configurations (especially for triangles) have been studied by many authors: the dynamic geometry generated by the incircle and the circumcircle of a triangle, the pedal triangle, the orthic triangle, and the incentral triangle.

A natural problem is to determine all real (or complex) numbers α for which the sequence $A_nB_nC_nD_n$, $n \geq 0$, is convergent. For triangles, Andrica, Bagdasar and Marinescu proved that the sequence converges if and only if $0 \leq \alpha \leq 1$, also providing the order of convergence. Also, Andrica and Bagdasar proved that for complex values of α, the sequence is convergent when this parameter is located within the interior of the intersection of two disks centered in two points related to the eigenvalues of the iteration matrix of the system. Also, the orbit is divergent when α is in the exterior of this region, while being periodic in four points. For certain values, the orbit may have a finite number of convergent subsequences, while in other cases the orbit can be dense within certain circles.

In this paper we investigate the geometry of sequence $A_nB_nC_nD_n$, $n \geq 0$ when α is a complex number. We prove that the sequence is convergent when α is in the interior of the intersection of the disks $D_1 \left(\frac{1}{2}, \frac{1}{2} - \frac{1}{2}i \right)$ and $D_3 \left(\frac{1}{2}, \frac{1}{2} + \frac{1}{2}i \right)$. We also investigate where the sequence diverges (exterior of this intersection) or periodic ($\alpha = 0$ or $\alpha = 1$). Parametrisations of the lower and upper boundaries of this domain allow us to identify when the sequence is dense or has a finite set of accumulation points.
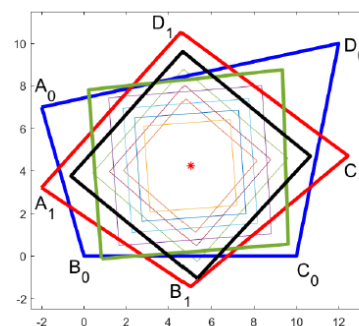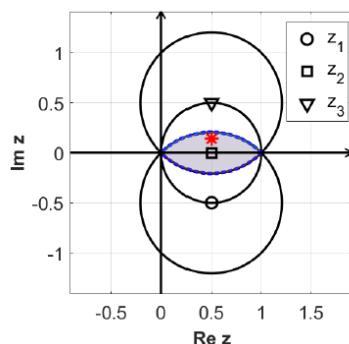
NEW RESULTS ON POSITIVE PERIODIC SOLUTIONS FOR A VECTOR-TRANSMITTED DISEASE MODEL WITH ITERATIVE TERM

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EXTENDED ABSTRACT

In this work, by using Krasnoselskii's and Banach fixed point theorems with Green's functions method we study the existence and uniqueness of positive periodic solution of the first order iterative vector-transmitted disease model. Our findings add to those already published in the literature.

CONCLUSIONS

This work is concerned with studying the existence and uniqueness of a first-order iterative differential equation.

Our most important step is to choose the appropriate space and subset for this study, followed by converting our equation into an equivalent integral equation, then utilizing Krasnoselskii's fixed point theorem as well as some properties of a Green's function for establishing the existence of the considered problem. While, for the uniqueness of the solution we used Banach contraction principle.

NEW PROOFS OF THE THEOREM
$$\text{rank } f(M) + \text{rank } g(M) = \text{rank}(f,g)(M) + \text{rank}[f,g](M)$$

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EXTENDED ABSTRACT

We show that the sum of the ranks of two matrix polynomials is the same as the sum of the rank of the matrix obtained by applying the greatest common divisor of the polynomials, with the rank of the matrix obtained by applying the least common multiple of the polynomials, i.e., for any two polynomials $f, g \in C[X]$ and for any matrix $M \in Mn(C)$, the following relation holds

$$rank\ f(M)\ +\ rank\ g(M)\ =\ rank\ (f,g)\ (M)\ +\ rank\ [f,g]\ (M).$$

Three new proofs of the above theorem are presented. Many applications, for older or more recent problems, of this result can be obtained. An important result, used in the first three proofs, is the following (see J.J. Rotman, Advanced Modern Algebra, Prentice Hall, 2003, Theorem 3.31, p. 135): *If K is a field, $f, g \in K[X]$, and d is their greatest common divisor, then there are $\varphi_1, \varphi_2 \in K[X]$ such that*

$$f \cdot \varphi_1\ +\ \varphi_2 \cdot g\ =\ d.$$

First proof uses the method of elementary transformations in block partitioned matrices. The second proof uses linear applications, reducing the equality to be demonstrated in the study of the equality case in Sylvester's inequality. The third proof reduces the equality to be demonstrated in the study of the equality case in Frobenius inequality, equivalent (see Y. Tian, G.P.H. Styan, When does $\text{rank}(ABC) = \text{rank}(AB) + \text{rank}(BC) - \text{rank}(B)$ hold?, International Journal of Mathematical Education in Science and Technology, 33, 2002, pp. 127–137) to the existence of two matrices, X and Y, such that $BCX + Y AB = B$. The fourth proof uses the Jordan's canonical form of a matrix. More precisely, it is shown that the equality to be demonstrated can be reduced to the case for which M is a Jordan cell.

Among the applications, the last two stand out, which are similar to the Inclusion–Exclusion Principle from set theory.

ON THE STABILITY OF A PREDATOR–PREY SYSTEM OF DIFFERENCE EQUATIONS

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EXTENDED ABSTRACT

Using the centre manifold theory, we study the stability of the zero equilibrium of a system of two difference equations in the special case when one of the eigenvalues is equal to 1 and the other eigenvalue has the absolute value less than 1. The proposed system describes the dynamics of a single two-stage population that is practicing cannibalism. The difference equations and the systems of difference equations, especially those involving exponential terms, have many applications in biology, population dynamics, genetics, economy, physics and other applied sciences. We study the stability of the zero equilibrium of the following system of difference equations

$$(1) \quad \begin{cases} x_{n+1} = \dfrac{y_n}{py_n + e^{-y_n}} + ax_n e^{-c_1 y_n} \\ y_{n+1} = b\, x_n e^{-c_2 y_n} + (1 - d)y_n. \end{cases}$$

where $p > 1, a > 0, b > 0, a + b < 1, 0 < d < 1, c1 \geq 0, c2 \geq 0, b = d(1 - a)$, and the initial values x_0 and y_0 are non-negative real numbers.

We also consider $\alpha = 1 - a$, $\beta = -d$,

$$\eta = \frac{\alpha(\alpha^2 (p - 1) + a\alpha c_1 - bc_2)}{(1 - \lambda_2)(\beta - \alpha)}, \qquad A = \frac{2\alpha(bc_2 + \alpha\beta - p\alpha\beta - a\beta c_1)}{\beta - \alpha},$$

and

$$B = \frac{3}{\beta - \alpha}(2p^2\alpha^3\beta - 4p\alpha^3\beta - 4\eta p\alpha\beta^2 + \alpha^3\beta + a\alpha^2\beta c_1^2 - b\alpha^2 c_2^2 + 4\eta\alpha\beta^2$$
$$- 2a\eta\alpha\beta c_1 + 2b\eta\alpha c_2 - 2a\eta\beta^2 c_1 + 2b\eta\beta c_2).$$

The main result of the paper states the following:
  (1) if $A = 0$ and $B > 0$, then the zero equilibrium of (1) is unstable;
  (2) if $A = 0$ and $B < 0$, then the zero equilibrium of (1) is asymptotically stable.

DIRECTIONAL DERIVATIVES AND SUBDIFFERENTIALS FOR SET-VALUED MAPS WITH APPLICATIONS IN SET OPTIMIZATION

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EXTENDED ABSTRACT

We present a general method to devise directional derivatives and subdifferentials for set-valued maps that generalize the corresponding constructions from the classical situation of real-valued functions. We show that these generalized differentiation objects enjoy some properties that, on the one hand, meaningfully extend the aforementioned case and, on the other hand, are useful to deal with the so-called ℓ-minimality in set optimization problems.
The presentation follows the work done in the papers [1] and [2].

CONCLUSIONS

The generalized differentiation objects for set-valued maps we introduce in this work seem to have a promising potential in dealing with set optimization problems in a more direct manner than other approaches in literature. We illustrate this point of view by showing several basic properties and consequences that are naturally expected from such kind of constructions, and from this perspective the present study is a promising one. Now, the subsequent question is to what extent these methods and ideas could be successful in order to cover other topics related to set optimization and/or could be explored in other directions. This is an open theme that we intend to pursuit in future research.

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ON HORADAM SEQUENCES WITH DENSE ORBITS AND PSEUDO-RANDOM NUMBER GENERATORS

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Key words: *Horadam sequences, pseudo-random number generator*

EXTENDED ABSTRACT

Horadam sequences are second order recurrences in the complex plane, de- pending on a family of four complex parameters (two initial values and two recurrence coefficients). These sequences have been investigated in depth over more than 60 years, but many new properties and applications are still being discovered. Small variations in the parameters have dramatic impact on the sequence orbits, which exhibit a wide variety of patterns: periodic, convergent, divergent, or dense within one dimensional curves. In this paper we prove that for specific parameter values, the sequence orbit is dense within an annulus, while the complex argument is uniformly distributed in an inter- val. We use this feature to design a pseudo-random number generator, which we test using Monte Carlo estimations. This algorithm is found to perform well against commonly used generators like Multiplicative Lagged Fibonacci and Mersenne Twister. We also discuss further extensions involving complex linear recurrent sequences or higher order.

LYAPUNOV FUNCTIONS FOR RANDOM ONE-SIDED DYNAMICAL SYSTEMS

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EXTENDED ABSTRACT

This paper considers tempered exponential stability and tempered exponential instability concepts for one-sided discrete-time random dynamical systems. These concepts are generalizations from the deterministic case. Using this, characterizations in terms of Lyapunov functions respectively Lyapunov norms are presented. Also, an approach in terms of considered concepts for the inverse and adjoint random discrete-time systems is derived.

TOPOLOGICAL INDICES - APPLICATION OF GRAPH THEORY IN CHEMISTRY

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Key words: *molecular, graph, psychedelic phenethylamines, topological, indices.*

EXTENDED ABSTRACT

A molecular graph is a mathematical representation of a molecule using graph theory. The typical notation used is $G=(V,E)$, where V is the set of vertices corresponding to the atoms of the compound and E is the set of edges corresponding to chemical bonds.

Molecular descriptors are numerical representations of the physical and chemical properties of substances. From a molecular graph, an important series of molecular descriptors, such as the topological indices, can be calculated. These descriptors have been successfully used for the characterization and then the identification of the class identity of various drugs of abuse.

In order to assess to what extent these descriptors are useful for an efficient detection of the 2C-x class of psychedelic amphetamines, an exploratory analysis based on Principal Component Analysis (PCA) was performed by using *The Unscrambler X* software. The input was formed by using a set of 6 positives (2C-x compounds), and 10 negatives representing a large variety of compounds of forensic interest. These compounds were characterized by computing, for each of them, a number of 79 topological descriptors by using the *alvaDesc* software.

The score plot displayed in Figure 1 indicates that the compounds belonging to the 2C-x class form a dense cluster, while the negatives are spread all over the plot.
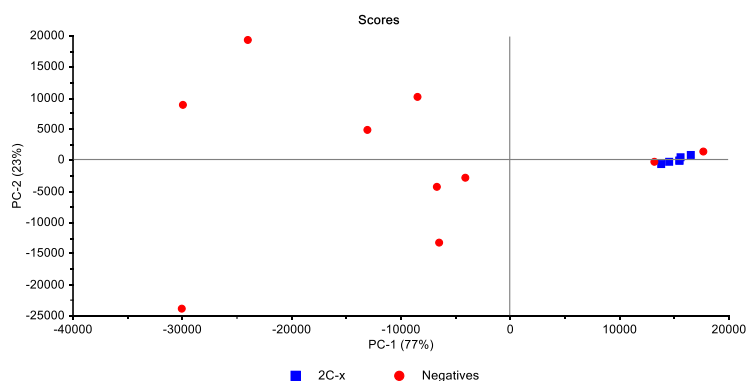
Figure1: Score plot indicating the cluster formed by the compounds belonging to the 2C-x class of psychedelic amphetamines

CONCLUSIONS

Topological indices provide the connection between mathematics and chemistry. The exploratory analysis performed based on PCA indicates that they can be a powerful input for building automated applications for the classification of drugs of abuse based on multivariate statistical methods.

ON DIRECTIONAL SUBREGULARITY OF MAPPINGS

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EXTENDED ABSTRACT

In this paper we present directional regularities and subregularities of mappings with respect to fixed sets. We establish a result concerning the directional subregularity of compositions, and as consequence we deduce a directional variant of Lim's Lemma.

STABILITY ANALYSIS OF IMPULSIVE STOCHASTIC LINEAR DIFFERENTIAL EQUATIONS

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EXTENDED ABSTRACT

In this talk we consider systems modelled by impulsive stochastic linear differential equations of Ito type. Several kinds of stability concepts are considered (strong exponentially stable in mean square, exponential stable in mean square, L2 stochastic stable in mean square, asymptotic stable in mean square) of the state equilibrium and we shall provide necessary and sufficient conditions which guarantee the desired kind of stability.

ESSENTIALLY SELFADJOINT LINEAR RELATIONS IN HILBERT SPACES

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EXTENDED ABSTRACT

A central problem in the operator theory is to describe and characterize selfadjoint and essentially selfadjoint linear operators in real or complex Hilbert spaces. These types of linear operators have many applications in quantum mechanics and in the theory of differential equations. However, there are situations when the underlying linear operator which models a physical phenomenon is not densely defined or is multi--valued. Hence, the theory of linear relations (multi--valued linear operators) enters into the play.

Let H be a real or a complex Hilbert space and let A be a linear relation in H, i.e. a linear subspace of the Cartesian product H x H. Then A^*, the adjoint of A, is a closed linear relation in A. The linear relation A is said to be essentially selfadjoint if $A^* = clos\, A$, where $clos\, A$ stands for the closure of A.

Essentially selfadjoint linear relations can be defined on real or complex Hilbert spaces. However, there are differences in treating essentially selfadjoint linear relations on real Hilbert spaces versus on complex Hilbert spaces.

The main objective of this paper is to provide some range matrix--type criteria for the essentially selfadjointness of symmetric linear relations in real or complex Hilbert spaces, respectively.

The paper is organized as follows. Section 1 contains basic notions concerning linear relations in Hilbert spaces. Some technical results are collected in Section 2. Certain characterizations for the essentially selfadjointness of symmetric linear relations in real or complex Hilbert spaces are presented in Section 3. Nonnegative essentially selfadjoint linear relations are treated in Section 4. Finally, certain range matrix--type criteria are proven in Section 5.

The techniques used for the proofs are specific to the case of linear relations and the obtained results are valid for both real and complex Hilbert spaces.

CONCLUSIONS

The main objective of this paper is to provide some range--type criteria for the essentially selfadjointness of symmetric linear relations in real or complex Hilbert spaces. The main used ingredient is a matrix whose entries are certain linear relations.

ADJOINT TO EACH OTHER LINEAR RELATIONS

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EXTENDED ABSTRACT

The present paper is inspired and motivated by the paper of Z. Sebestyen and Zs. Tarcsay, *Adjoint of sums and products of operators in Hilbert spaces, Acta Sci. Math.* (2016).
It can be viewed as a natural continuation of A. Sandovici, *On the Adjoint of Linear Relations in Hilbert Spaces*. Mediterranean J. Math. (2020) and the question to ask is the same, namely find necessary and sufficient conditions for two linear relations to be adjoint to each other.

First results are obtained using some purely algebraic relations concerning the main objects associated with a linear relation. Criteria for a linear relation to be adjoint to another one are offered in terms of their domains and multivalued parts, and by duality, in terms of their ranges and kernels, respectively.

In the latter part of the paper, a matrix linear relation will enter into the play. It will be in conjunction to the resolvent operator associated in a certain way to the pair of linear relations *A* and *B*.

A remarkable advantage of the present treatment is that the results are not limited to complex Hilbert
spaces; they will remain valid in real Hilbert spaces as well. As direct consequences new characterizations for closed, skew--adjoint, and selfadjoint linear relations are obtained.

CONCLUSIONS

The results of the present paper generalize to the case of linear relations certain results obtained by D. Popovici, Z. Sebestyen and Zs. Tarcsay in their papers and also by T. Nieminen.

ON THE CLOSEDNESS AND THE SELFADJOINTNESS
OF THE SUM OF TWO LINEAR RELATIONS

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Key words: *Hilbert space, closed linear relation, selfadjoint linear relation, essentially selfadjoint linear relation.*

EXTENDED ABSTRACT

The main objective of this paper is to provide some criteria for the closedness and the self--adjointness of the sum of two linear relations in real or complex Hilbert spaces, respectively; for more information and related works see [5, 6] and the references therein.

The paper is organized as follows. Section 1 contains basic notions concerning linear relations in Hilbert spaces. Some algebraical results of technical nature are collected in Section 2. New criteria for the closedness and the selfadjointness of the sum of two linear relations in Hilbert spaces are provided in Section 3. Finally, in Section 4 two perturbation type results are stated and proven.

The techniques used for the proofs are specific to the case of linear relations and the obtained results are valid for both real and complex Hilbert spaces.

CONCLUSIONS

The material presented in this paper generalize some results in [2] and [3]. It is strongly connected to results in [1, 4, 5, 6].

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NEW RESULTS FOR ANALYTIC FUNCTIONS

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EXTENDED ABSTRACT

In this paper we present some new results and propose some cutting-edge results for the specialists in the field of geometric function theory.

A COMPARATIVE STUDY ON THE CONTRACTIVE OPERATORS

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Key words: classical contractive operators, Bannach operator, Kannan operator, contractions, theorem of fixed point.

EXTENDED ABSTRACT

This paper aims to present a comparative study between contractive operators Bannach and Kannan. There are highlighted the primary conditions which must be fellfield for contractions and introduced representative examples for cases: Kannan is contraction, Kannan is not contraction and the contraction which is not Kannan.

CONCLUSIONS

Kannan found a particular type of operators which are not necessarily continuous, but satisfies the fixed point property on complete metric spaces. The class operators found by Kannan and that of Bannach are independent of each other. A new extension of Kannan's fixed point theorem has applications to integral equations, it is also useful for split feasibility and variational inequality problems. So, the contractive operators presented are efficient for a large field of applied mathematics and computer science in engineering, in the previous sentence I presented a few domains for their applicability.

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OPTIMAL JOB SCHEDULING USING BIPARTITE GRAPHS

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EXTENDED ABSTRACT

This paper aims are to analyze different situations in which we are forced to make a decision based on our current options and to reinterpret it as a bipartite graph (bigraph) and to approach it as an assignment problem. We also want to introduce a new algorithm based on the Hungarian method to solve those abovementioned problems. Starting from an idea found in the paper [6] we managed to come up with a new modified Hungarian algorithm (MHA) that can solve most assignment problems. All the problems solved with the Hungarian algorithm assume a *square matrix*, that is $n \times n$ dimensions. In real life situations this is rarely the case, and the dimensions of the matrix are $n \times m$, where $n \neq m$. This can provide a simpler way to find a solution in areas where that solution could be critical. One situation that we will discuss is how can we distribute tasks to a number of machines/people such that the total required time to complete those tasks is minimum or maximum, job scheduling requires an optimal distribution of workload to obtain the desired solution.

CONCLUSIONS

We have come to the conclusion that by illustrating different real-life situations as bipartite graphs we can more easily think of a solution. Also, the modified Hungarian algorithm that we introduced can have a great impact today since a lot of people are struggling to optimally distribute resources around the world. Taking everything into account, we are also looking for ways to limit how many resources can be assigned to one place and instead distribute them elsewhere.

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MINIMAX ALGORITHM FOR OPTIMAL JOB ASSIGNMENT

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Key words: *game theory; minimax method; chosen criteria; optimal job assignment.*

EXTENDED ABSTRACT

This paper aims to present the minimax method based on game theory, applied to optimal job scheduling. In this regard, the minimax algorithm consists in the selection of the potential candidates by the companies according to several criteria, for example the specialization, the grades gotten that matters for their applicability within the company and eventually their experience. The algorithm can generate a number of solutions, leading to several options for the employer, if the solutions coincide with the chosen criteria, so the next step can be the interview, else continuation of the algorithm through a defining criterion.

CONCLUSIONS

A reconfigurable filter bank for digital hearing aid in Canonical Sign Digit Space is proposed to reduce the complexity while implementing hardware. The gain of each subband is then optimized using Minimax algorithm. The proposed filter bank can achieve satisfactory performance in audiogram matching which is shown by means of examples.

Gradient Feature Weighted Minimax (GFW Minimax) method was developed to register multiple portal images to three-dimensional CT images. Minimax algorithm was significantly more accurate and robust.

Adaptive strategy for external beam radiation therapy in prostate cancer: management of the geometrical uncertainties with robust optimization : First developed for protons, the minimax robust optimization method implemented by Frediksson allows the sampling of the setup uncertainties into multiple and provides a solution that minimizes the objective function such that the prescription is satisfied even in the worst-case scenario, with the potential to reduce the dose to healthy tissue

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A HYBRID ALGORITHM BASED ON THE OPTIMAL PATH WITH APPLICATIONS IN NAVIGATION SYSTEMS

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Key words: *Bellman-Ford, Dijkstra; Hybrid algorithm; navigation systems; optimal route;*

EXTENDED ABSTRACT

In literature, there is a special class of algorithms (e.g., Bellman Ford and Dijkstra) that determine the shortest path from a single source, each calculating the shortest distance from the source node to other nodes of the graph considered. However, there exists significant differences between them, specific advantages, and disadvantages, such as computation time, the Greedy method used by Dijkstra not always providing the best solution, in some situations negative values for the edges cannot be used, etc. Given the particularities of each, the main idea of this paper is designing a hybrid algorithm with applications in any navigation system that determines the optimal route from one point to another point or close variations of the most efficient, so that the computational time is minimal. The goal of the hybrid Bellman-Ford-Dijkstra algorithm is to improve the running time of graphs that contain negative values. Dijkstra is used several times without re-initializing the value at nodes, speeding up the process. The algorithm uses both Bellman-Ford and Dijkstra in unison to find the shortest path from a source node in a graph with general edge costs.

CONCLUSIONS

By combining the two shortest path algorithms, we can utilize them to the best of their abilities, combining their strengths and overcoming their weaknesses. The hybrid Bellman-Ford-Dijkstra algorithm incorporates Dijkstra rounds without re-initializing the values of other vertices resulting in a better running time and also being able to handle negative values. The algorithm could still be improved upon using other algorithms, aiming for the best results in the shortest time. In applications, it could be adapted to show the shortest path (in terms of cost), the optimal path (in terms of time), and keep track of other factors.

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UNIFORM STABILITY CONCEPTS FOR DISCRETE-TIME SKEW-EVOLUTION COCYCLES

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Key words: *uniform stability; uniform exponential stability; uniform polynomial stability; discrete-time skew-evolution cocycles.*

EXTENDED ABSTRACT

In this paper we present the concept of uniform stability for dynamical systems described by discrete-time skew-evolution cocycles in Banach spaces. We emphasize two concepts of uniform stability as uniform exponential stability and uniform polynomial stability. Different characterizations and connections between the above concepts are established.

CONCLUSIONS

The main purpose of this paper is to give some generalizations of the classical results of Barbashin and Datko-type for characterizations of uniform stability concepts (exponential and polynomial) in the case of discrete-time skew-evolution cocycles in Banach space.
In the future, the authors would like to study the variants of these results in the nonuniform case.

A NEW GENERALIZATION OF r-STIRLING NUMBERS OF THE FIRST KIND

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EXTENDED ABSTRACT

For all integers $n, k \geq 1$, let $\begin{bmatrix} n \\ k \end{bmatrix}$ be the classical unsigned Stirling number of the first kind. This number is defined as follows: $\begin{bmatrix} n \\ k \end{bmatrix}$ is the number of permutations of $[n] := \{1, 2, \ldots, n\}$ into k non- empty cyclically ordered sets, i.e. cycles. One of the most interesting generalization of Stirling numbers was the r-Stirling numbers introduced By Broder [2]. Analogously to the classical Stirling numbers, the author considered that r-Stirling numbers of the first kind $\begin{bmatrix} n \\ k \end{bmatrix}_r$ count the number of all permutations of $[n]$ having exactly k cycles such that the r first elements $1, 2, \ldots, r$ lead.

We propose a new generalization of the r-Stirling numbers of the first kind and their analogues. These numbers appear as specialization of a new class of symmetric function [1], and they can be seen as a natural generalizations of the r-Stirling numbers of the first kind and their analogues. We also give combinatorial interpretations of the classical numbers in terms of s-tuples of permutations of $[n]$ with k cycles where the first r elements of each permutation are in distinct cycles, and using the inversion and the co-inversion statistics on the cycles in the cases of the analogues numbers. Moreover, using the hyperharmonic numbers and the r-Stirling numbers of the first kind new formulas and useful properties are established.

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STABILITY ANALYSIS IN A NEW MODEL FOR DESENSITIZATION OF ALLERGIC REACTIONS INDUCED BY CHEMOTHERAPY OF CHRONIC LYMPHOCYTIC LEUKAEMIA

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Key words*: Allergies, Drug desensitization, *Chronic Lymphocytic Leukaemia*, Delay-differential equations, Stability Analysis.*

EXTENDED ABSTRACT

INTRODUCTION

We describe a competitive model that reflects the cellular evolution of patients with chronic lymphocytic leukaemia who are receiving chemotherapy.Due to drug allergic reactions associated with chemotherapy, we have incorporated in the model the desensitization for the allergies caused by the chemotherapy. The system is modelled using delay differential equations. The immune system's function, as well as the chemotherapeutic involvement in cancer cure and the allergic reactions caused by the therapy, are further explored using the classical linear stability analysis and numerical simulations.

First, we introduce and thoroughly present the mathematical model, then we study some relevant properties of the solutions (Positivity, boundedness and global existence). We investigate biologically significant equilibrium points and their stability and finally we present numerical simulations and offer biological interpretations.

RESULTS

We proposed a mathematical model which describes the evolution of T helper cells, APCs and cytokines in the case of drug allergies with exposure to drug desensitization during chemotherapy of chronic Lymphocytic Leukaemia. The novelty of the model consists in a more detailed approach of the biological mechanism.

Another important improvement from the existing literature is the presence of time delays. These help to better capture the correct timeframe of the biological processes.

From a mathematical point of view, existence, boundedness, positivity and stability of solutions are studied. The positivity of solutions is an essential study to undertake, as we are modelling cell populations. We also give parameter conditions for stability in the case of some equilibrium points.

The stability of the equilibrium points in question illustrates the benefits of drug desensitization. The numerical simulations we performed validate the theoretical results, basically showing that a small amount of allergen does not do any harm in chemotherapy of CLL.

In our simulations we also find that the Th2/Th1 ratio decrease during chemotherapy in both equilibrium points (E1 & E2), which means that concentration of Th1 dominate the concentration of Th2 during chemotherapy. This might be an explanation that small quantities of chemotherapeutic drug (i.e., by desensitization) will not cause an allergic reaction.

ABOUT ISIM & ISWIM 2023 – 2nd edition

The University POLITEHNICA of Bucharest, Center for Research and Training in Innovative Techniques of Applied Mathematics in Engineering "*Traian Lalescu*" (CiTi) in collaboration with "1 DECEMBRIE 1918" University of Alba Iulia are honored to invite you to participate in the 2nd Edition of the **International Symposium & International Student Workshop on Interdisciplinary Mathematics in the CiTi areas (ISIM & ISWIM)** that will be held at University Politehnica of Bucharest, Romania in June 2023.

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2. Data analysis and information security, game theory - including quantum and applications.

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4. **NEW!** Non-classical sets (algebric and topological aspects) and applications

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1. Applications of fractional calculus.

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